

The principle of maximum flow energy and the anomalous accelerations of spacecrafts

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Abstract

The anomalous delay of *Pioneer* 10, the unexpected increase of velocity during *Galileo*'s first positive gravity assist, and *NEAR*'s anomalous velocity during a negative gravity assist are examined through the lens of a hypothetical variational principle: maximum flow energy. Energy exchange processes within a volume, normally considered to have no physical consequences, are now taken into account as a possible source of fine corrections to Hamilton's principle. The motion of thermal energy tied up in the materials travelling through space is one source of energy flow, as is the exchange of energy arising from the motion of the spacecraft through an electromagnetic field. In all three cases the differential equation proposed as a solution produces estimates of the correct order of magnitude. However, there are two strong sources of error in these estimates: the lack of published details concerning the material composition and the temperature of the spacecrafts, and the graphical evaluations used to determine their time-dependent velocity during the gravity assists.

PACS: 95.10.Eg; 02.30.Yy; 41.20Jb; 65.40.Ba

Keywords: Planetary gravity assist; radiant heat

To date three anomalous motions have been well documented in spacecrafts: the unexpected negative acceleration experienced by *Pioneer* 10 on leaving the solar system (Anderson et al., 2002), the anomalous increase in *Galileo*'s speed during its first gravity assist by Earth, and a similar effect felt by *NEAR* (Antreasian and Guin, 1998). All await a physical explanation. It has been supposed that a common but unorthodox effect might be responsible (Lämmerzahl et al., 2006). As a most likely cause radiant heat has been favoured (Nieto and Anderson, 2007). Here I treat these problems by introducing a hypothetical variational principle: maximum flow energy. This concept has proven useful in explaining ordering phenomena in the liquid and gaseous states (Eidenschink, 2007).

This principle, comprising also Hamilton's principle of least action, can be expressed classically as

$$\int_V \int_t \Pi dV dt = \max. \quad (1).$$

V is the volume in which a process takes place, t is the time, and Π is the energy exchanged per unit time. Π has the dimensions of power per volume. In terms of quantum mechanics, exchange refers to any change in the quantum state and/or position of a particle. Equation (1) also includes exchange processes that in physics and physical chemistry are regarded as imaginary, e.g. elastic collisions between molecules and the electromagnetic flux defined by Poynting's theorem. As all past attempts at explaining the anomalous motions have been based on Hamilton's principle – i.e., by Newton's axioms – I shall apply (1) to imaginary processes only not accounted for in the principle of least action. Specifically, I will focus on the movement of a material object and its heat content through an electromagnetic field originating from outside or inside the spacecraft.

Pioneer 10 far beyond Saturn

This quantity Π is related to the movement of the spacecraft's constant volume V_S in two respects:

1. Transported exchange energy. Within the spacecraft, a certain amount of energy E_E is steadily being exchanged. Here, this energy is taken to be equal to the enthalpy or heat content ($H_T - H_0$). If the spacecraft is passing through space at a constant velocity v the energy flow will be constant. It can be calculated by considering the motion of a small area element A with speed v through space, which yields $A \cdot \rho_E \cdot v$ where $\rho_E = E_E/V_S$ is a scalar.
2. The exchange of energy with the environment. At a distance r from the Sun the power density of radiation is $\rho_{Su} = L_{Su}/4\pi r^2 \cdot 1$, where L_{Su} is the solar luminosity. The Poynting vector of the alternating electromagnetic field from Sun passes through A moving at speed v . When the spacecraft has moved a distance s parallel to c the *imaginary* power exchanged *within* the volume segment of the spacecraft behind A is $+2 \cdot A \cdot \rho_{Su} \cdot s \cdot v/c$. The factor 2 expresses the fact that energy is taken up *and* released. The power density ρ_{Su} is treated a vector quantity linked dyadically with s . Its positive sign expresses the fact that the spacecraft is moving in the same direction as the electromagnetic field from its source.

Let Π_V be the power contained within the constant volume segment of the spacecraft that can be attributed solely to its velocity v relative to Sun:

$$\Pi_V = A \cdot \rho_E \cdot v + 2 \cdot A \cdot \rho_{Su} \cdot \left| \frac{v}{c} \right| \cdot s \quad (2).$$

An exact mathematical treatment of the variational problem (1) is lacking. To begin with, it may be assumed that the first derivative of Π_V – the integrand in (1) – with respect to time has to vanish; this condition permits a stationary solution to exist. Keeping v/c constant and neglecting the fact that ρ_{Su} decreases with distance, one obtains

$$\Pi'_V = 0 = \rho_E \cdot a + 2 \cdot \rho_{Su} \cdot \left| \frac{v}{c} \right| \cdot v \quad (3).$$

It has already been conjectured that particles moving away from Sun but still exchanging energy within the corona should experience a negative acceleration on the basis of equation (1) (Eidenschink, 1994).

Here, I can only check whether condition (3) is satisfied by published data. For *Pioneer 10*, an acceleration of $-8.7 \pm 1.3 \cdot 10^{-10} \text{ m} \cdot \text{s}^{-2}$ has been measured at a distance of 67 AU (1 AU $1.49 \cdot 10^8 \text{ km}$) from the Sun; its velocity is $12.2 \text{ km} \cdot \text{s}^{-1}$ (Anderson et al., 2002). There is some uncertainty about the value of ρ_E because the material composition and temperatures of the spacecraft are not fully known. For the platform, which is heated by ^{238}Pu radioisotope thermoelectric generators (RTG), the temperature range $220 \text{ K} > T < 355 \text{ K}$ has been published (Anderson et al. 2002). To obtain an order-of-magnitude estimate for ρ_E , assume that the spacecraft is made of pure aluminium and a mean T of 288 K. This implies a heat content ($H_T - H_0$) of $4.39 \cdot 10^3 \text{ Joule} \cdot \text{mol}^{-1}$ (Downie, Martin, 1980; Leadbetter, 1968). For a density of 2.69 and an atomic mass of 27, ρ_E is $4.29 \cdot 10^8 \text{ Joule} \cdot \text{m}^{-3}$. (For a temperature of 220 K one

obtains $2.17 \cdot 10^8 \text{ Joule} \cdot \text{m}^{-3}$, and for 355 K the result is $5.96 \cdot 10^8 \text{ Joule} \cdot \text{m}^{-3}$). As a rule, metals with higher atomic numbers than aluminium will still have higher values of $(H_T - H_0)$; and organic compound will have lower values (Tsuji et al., 1982). The solar luminosity $L_{Su}=3.85 \cdot 10^{26} \text{ W}$ at a distance of 67 AU gives $\rho_{Su}=0.307 \text{ Joule} \cdot \text{s}^{-1} \cdot \text{m}^{-3}$. Equation (3) yields an acceleration of $-6.9 \cdot 10^{-10} \text{ m} \cdot \text{s}^{-2}$, which is the proper order of magnitude.

The obvious question is why no higher values were detected when *Pioneer 10* was closer to the Sun. This may perhaps be explained by relatively big disturbances of the measurements occurring in this stage.

The *Galileo* positive gravity assist

The speed and direction of a spacecraft can be changed enormously by a gravity assist (Van Allen, 2003; Anderson, 2007). The full history of the anomalous accelerations observed in the two Earth flybys discussed here could not be verified due to technical circumstances. However, for *Galileo* it was established that an anomalous velocity increase of $3.9 \cdot 10^{-3} \text{ m} \cdot \text{s}^{-1}$ occurred on its first flyby within one hour of periapsis.

The essential question why such an effect has never been observed in natural objects has a surprisingly simple answer: all the spacecrafts mentioned here contain plutonium-oxide RTGs who's power output L_{Pu} is finally converted into heat. In the sense of principle (1), a significant contribution to Π_V arises from the enormous acceleration of the spacecraft in the gravitation field of the Earth. Let us now investigate how Π_V depends on the component of velocity parallel to Earth's movement.

Again, Π_V consists of two parts:

1. The energy $(H_T - H_0)$ exchanged within V_S now has a time-dependent velocity. In a geocentric coordinate system, the trajectory of the spacecraft is hyperbolic (Fig.1a, after Antreasian and Guin, 1998, shows *Galileo*'s trajectory distorted by projection onto the ecliptic). The corresponding expression for energy flow is $A \cdot \rho_E \cdot v_e$, where v_e is the component of the spacecraft's velocity parallel to Earth's trajectory in a heliocentric coordinate system.

Fig.1a

2. Energy exchange with environment must take into account the enormous acceleration of V_S towards the Earth's centre of gravity. The power density in V_S is $\rho_{Pu}=L_{Pu}/V_S$. When *Galileo* has travelled an *additional* distance s_t due to its gravitational acceleration, the imaginary power is $2 \cdot A \cdot \rho_{Pu} \cdot v_m / c \cdot s_t$. For v_m I take the arithmetic mean of *Galileo*'s heliocentric velocities 1 hour before and 1 hour after periapsis (v.i.). The factor v_m/c gives the part of the radiation power thought to be run through by the movement of the spacecraft. Within the frame of this concept the spacecraft is assumed to move out of its own field of radiation. In the case of a uniform velocity, as with *Pioneer 10*, ρ_{Pu} does not play a role.

Thus, Π_V can be expressed by

$$\Pi_V = A \cdot \rho_E \cdot v_e + 2 \cdot A \cdot \rho_{Pu} \cdot \left| \frac{v_m}{c} \right| \cdot s_t \quad (4).$$

In analogy with (3), one can write:

$$\Pi'_V = 0 = \rho_E \cdot dv_e / dt + 2 \cdot \rho_{Pu} \cdot \left| \frac{v_m}{c} \right| \cdot u_t \quad (5)$$

The term u_t is formally an additional velocity caused by gravitation. It follows that

$$\int_{t_0}^{t_e} dv_e = \Delta v_e = -2 \cdot \frac{\rho_{Pu}}{\rho_E} \cdot \left| \frac{v_m}{c} \right| \cdot \int_{t_0}^{t_e} u_t \cdot dt \quad (6)$$

where t_0 and t_e shall be the (arbitrary) “onset” and “ending” times of the noticeable additional velocity term u_t .

I shall now try to support the underlying hypothesis using available data.

Again, to get an exact ρ_E one ought to know the materials present and their temperatures. Obviously, there are no published data of the temperature on board of *Galileo* at the time of its flyby. I therefore make the arbitrary choice of 288 K once again. Unlike *Pioneer 10*, *Galileo* had on board a considerable supply of dinitrogen tetroxide (N_2O_4) and methyl hydrazine ($H_2N-NHCH_3$) for future maneuvers. The total mass of the *Galileo* and the fuel at the time of launch are commonly known. Under the assumption that at the time of the first Earth flyby two thirds of the fuel was still on board, and that the „dry mass“ of the spacecraft was aluminium, one obtains the following numbers for the materials moving through space (mass in kg, atomic or molecular mass in Daltons, density in $g \cdot cm^{-3}$, heat content ($H_T - H_0$) in $Joule \cdot mol^{-1}$):
 Al (1575, 27.0, 2.69, $4.29 \cdot 10^3$),
 N_2O_4 (397, 92.0, 1.45, $3.63 \cdot 10^4$ (Giauque and Kemp, 1983)),
 $H_2N-NHCH_3$ (243, 46.1, 0.875, $2.71 \cdot 10^4$ (Aston et al., 1951)).

Under these assumptions, V_S can be estimated as $1.14 m^3$ and ρ_E as $4.75 \cdot 10^8$ $Joule \cdot m^{-3}$. Obviously, any inaccuracy of ρ_E depends more on the temperature than the details of the spacecraft's composition.

The electrical power produced by *Galileo*'s RTGs, with 21 kg ^{238}Pu on board, has been published as 560 W. The efficiency of its elements is estimated to be 6 %, giving a rough value for ρ_{Pu} of $8.2 \cdot 10^3$ $Joule \cdot m^{-3} \cdot s^{-1}$.

The time-dependent velocity per mass unit has been calculated for *Galileo*'s flyby from 3 h before to 3 h after its periapsis (closest approach) and published as a diagram (Anderson et al., 2007). The corresponding time-dependent velocities v_t relative to the Sun are plotted in Fig.1b.

Fig.1b

The angle φ_t between v_t and the trajectory of the Earth can be read out from a diagram (Antreasian and Guin, 1998) only roughly (the cosine function in Fig.1c).

Fig.1c

From Fig.1b, one can obtain Δv_t graphically for small time intervals. The time-dependent additional velocity u_t can then be calculated as $u_t = \Delta v_t \cdot \cos \phi_t$, the projection of Δv_t onto the Earth's trajectory.

Fig.1d

Fig.1d shows that the variation of u_t with respect to time is very strong near periapsis. Graphical integration of this curve yields $-1.3 \cdot 10^6$ m. The average heliocentric velocity v_m can be estimated from the velocities and angles given (Antreasian and Guin, 1998) according to the method described by Van Allan (2003). The result is $32.7 \text{ km} \cdot \text{s}^{-1}$. With the rough values already obtained for ρ_E and ρ_{Pu} , Δv_e is calculated as $4.9 \cdot 10^{-3} \text{ m} \cdot \text{s}^{-1}$. This is the same order of magnitude as the value obtained by Doppler measurement from Earth.

NEAR, negative gravity assist

The intended decrease of *NEAR*'s heliocentric orbital energy was brought about by a frontal approach to Earth. This flyby resulted in an anomalous increase of velocity as measured by radio Doppler: $13.5 \cdot 10^{-3} \text{ m} \cdot \text{s}^{-1}$. Fig.2a shows a projection of *NEAR*'s trajectory onto the ecliptic (after Antreasian and Guin, 1998).

Fig.2a

In Fig.2b, kinetic energies per unit mass were taken from a published curve (Anderson et al., 2007) and transformed to the velocity v_t .

Fig.2b

Fig.2c gives cosine of the angle ϕ_t between Earth's trajectory and the position vector of *NEAR* in the geocentric system, but these data are not precise because of the poor resolution of the graph and the strong distortion due to projection.

Fig.2c

Small intervals of Δv_t are taken from Fig.2b and multiplied by the corresponding cosine from Fig.2c to produce the function of $u_t = f(t)$ (Fig.2d).

Fig.2d

In analogy to (5) one can write

$$\Pi'_V = 0 = \rho_E \cdot (-dv_e / dt) + 2 \cdot \rho_{Pu} \cdot v_m / c \cdot u_t \quad (7).$$

The negative sign expresses the fact that in this gravity assist v_e is diminished. Graphic integration of the curve in Fig.2d yields $3.2 \cdot 10^6$ m. v_m can be estimated as $25.7 \text{ km} \cdot \text{s}^{-1}$ on the basis of the geocentric velocities and angles described above. There are no published data concerning mass and temperature of *NEAR* during its flyby. Also the heat generated by the RTGs is not available. If one takes as a first approximation the same values for ρ_E and ρ_{Pu} as in the case of *Galileo* Δv_e can be calculated as $9.5 \cdot 10^{-3} \text{ m} \cdot \text{s}^{-1}$, which again is the right order of magnitude.

Note that $u_i=f(t)$ near periapsis quickly changes between positive and negative slopes. Perhaps this is linked to the transient phenomenon of Δv_e observed as *NEAR* moved away from Earth (Antreasian and Guin, 1998).

Summary and Outlook

By the hypothetical principle of maximum flow energy, I have found the first coherent explanation for all three cases of anomalous spacecrafts motions on record. It is hoped that this approach will be examined and confirmed by those with additional unpublished data. Unfortunately, more accurate methods of evaluation exceed the means of the author. It is possible that the principle of maximum energy flow can be applied to other phenomena observed in space.

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Figures and legends

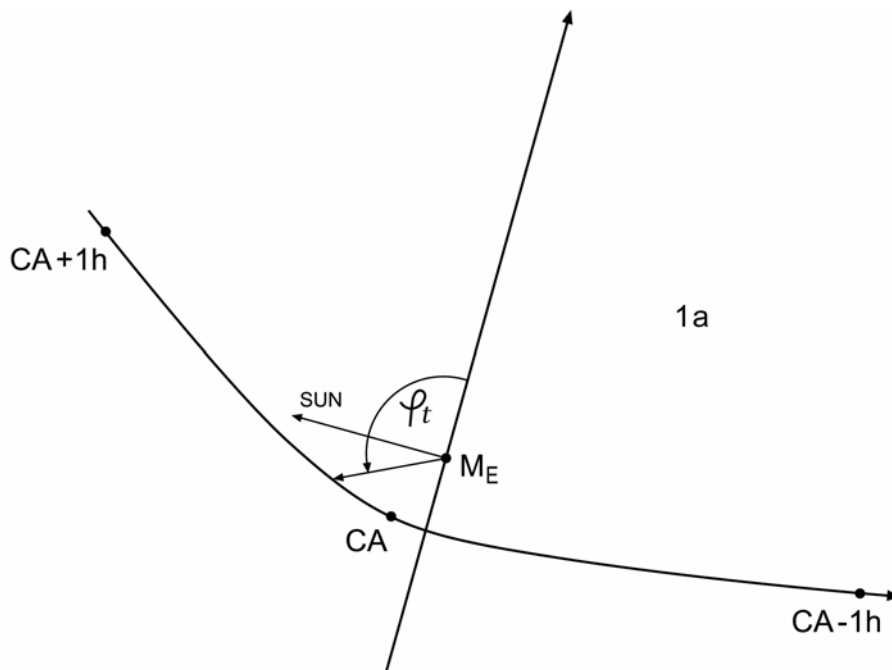


Fig.1a. Earth flyby of GALILEO on 8 Dec. 1990. The curve is a projection of the craft's trajectory onto the ecliptic, from ca. 1 h before to 1 h after periaresis (closest approach to the center of mass M_E). Also shown is φ_t the time-dependent angle between Earth's trajectory and the position vector of the spacecraft.

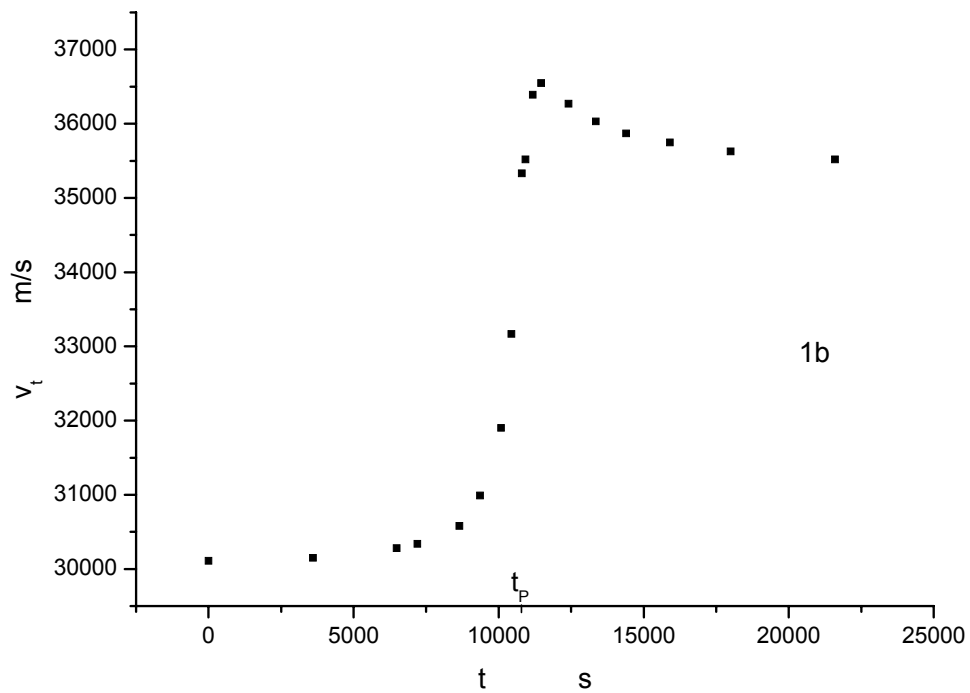


Fig.1b. Velocity v_t of *Galileo* from 3 h (10800 s) before to 3 h after periaresis (t_P).

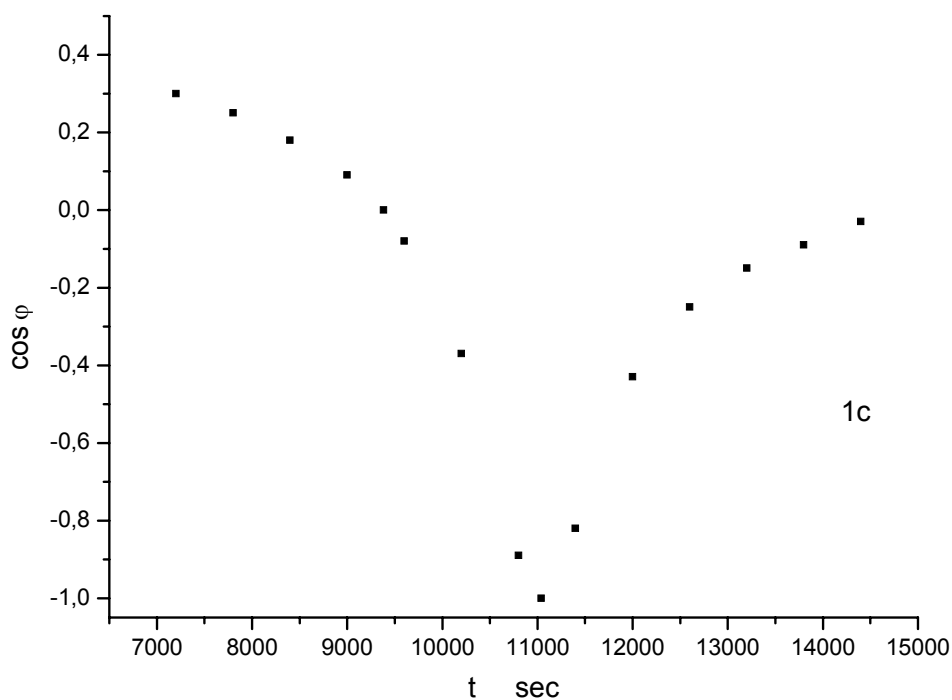


Fig.1c. Cosine(ϕ_t) of *Galileo* vs. time, using the narrowed scale of Fig.1b.

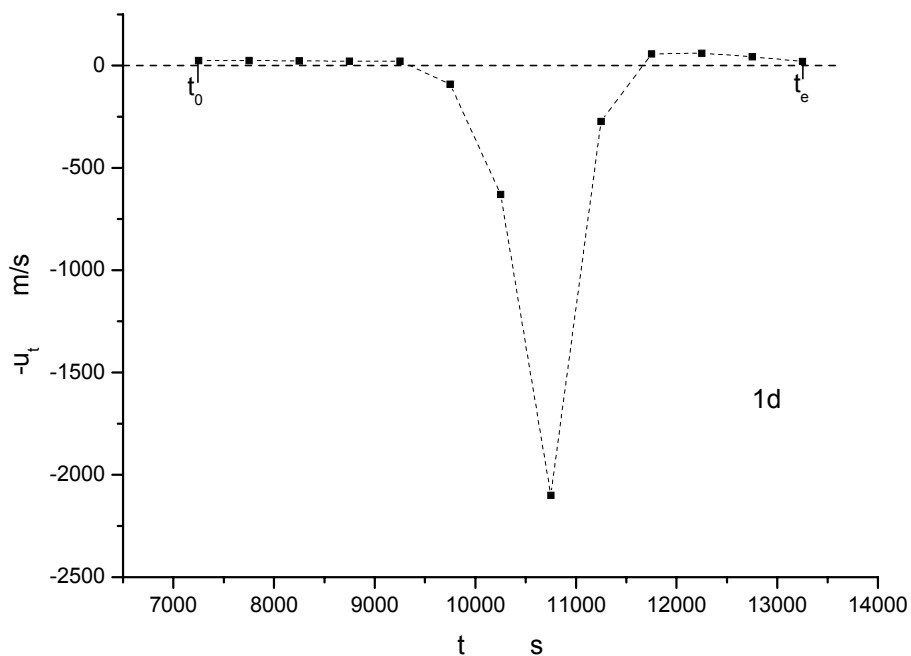


Fig.1d. Additional velocity u_t of *Galileo* parallel to the trajectory of Earth. The times t_0 and t_e , marking the limits of integration, are indicated to the unsmoothed curve. This figure uses the narrowed timescale of Fig.1b.

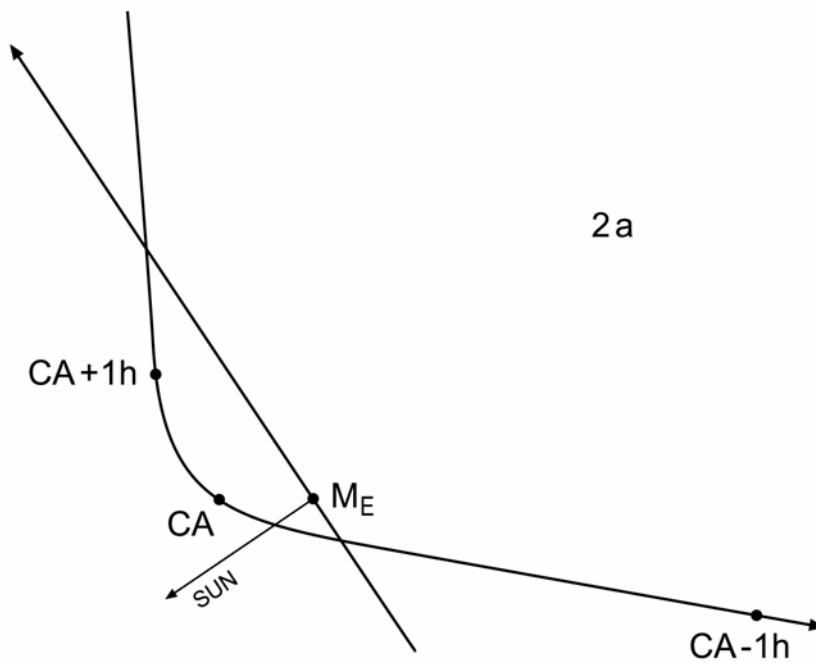


Fig.2a. Earth flyby of *NEAR* on 23 Jan. 1998; the curve is a projection of its trajectory onto the ecliptic.

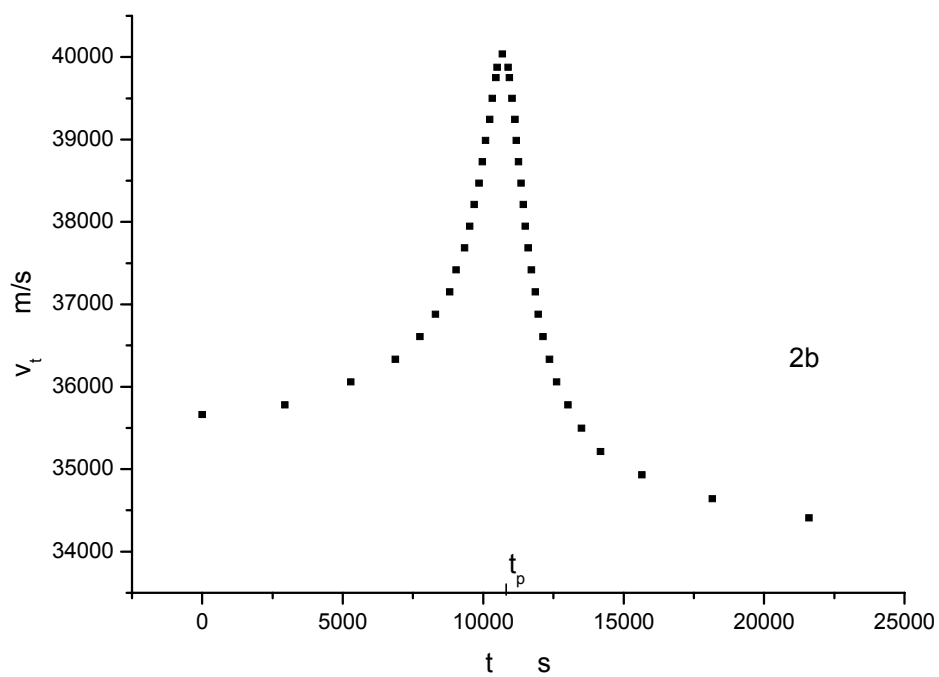


Fig.2b. Velocity of *NEAR* from 3h (10800 s) before to 3 h after periapsis (t_p).

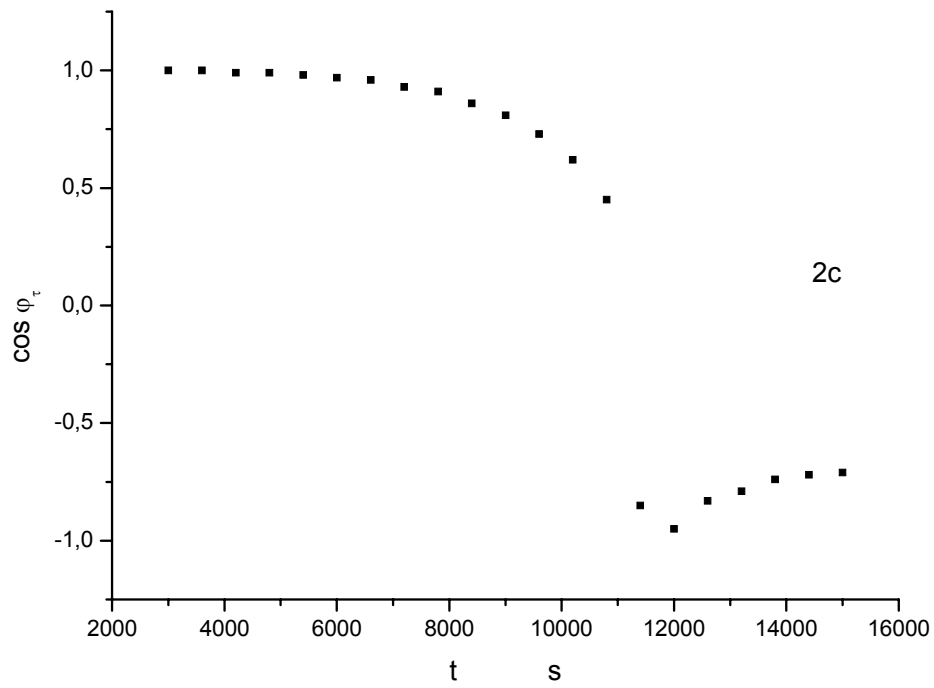


Fig.2c. Cosine(ϕ_t) of *NEAR* vs. time, using the narrowed timescale of Fig.2b.

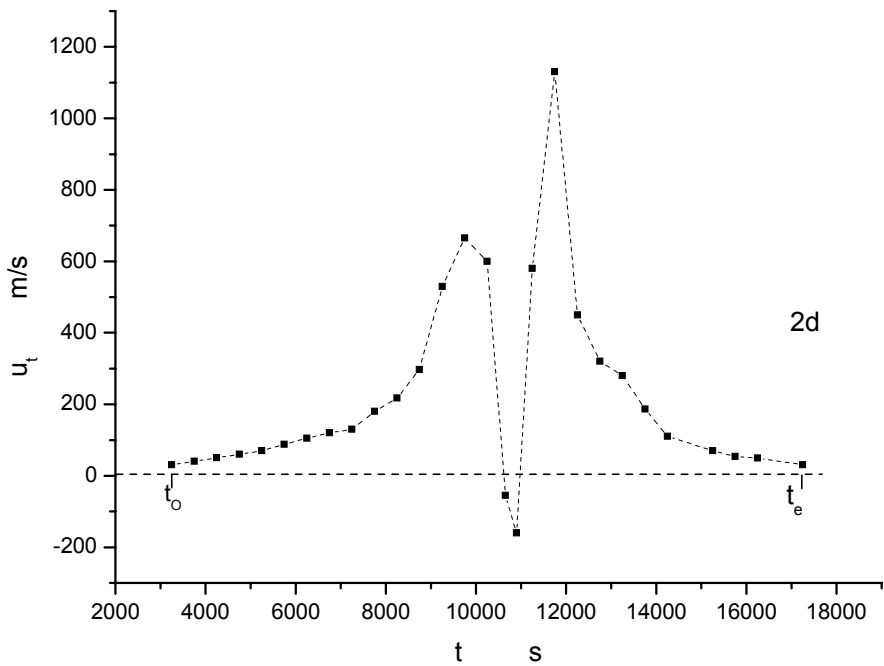


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